CIRCUITS, LOWER BOUNDS, AND CIRCUIT ANALYSIS ALGORITHMS

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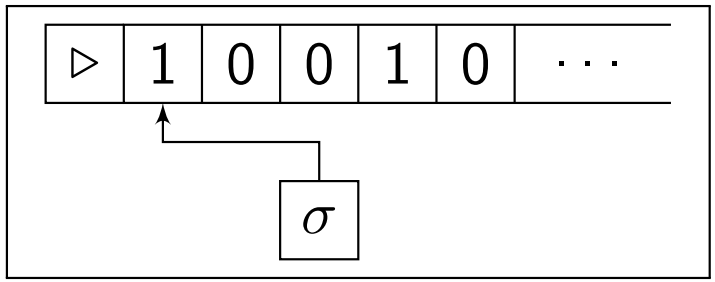
Introduction

$$f : \{0,1\}^n \to \{0,1\}$$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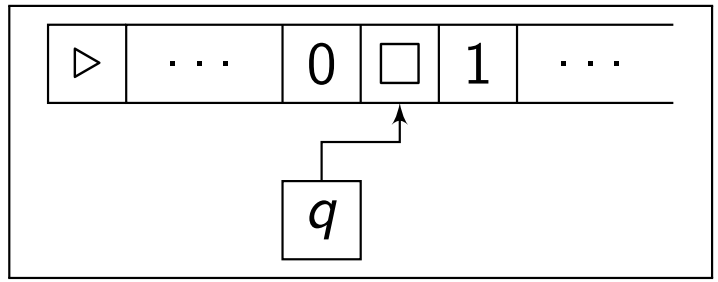
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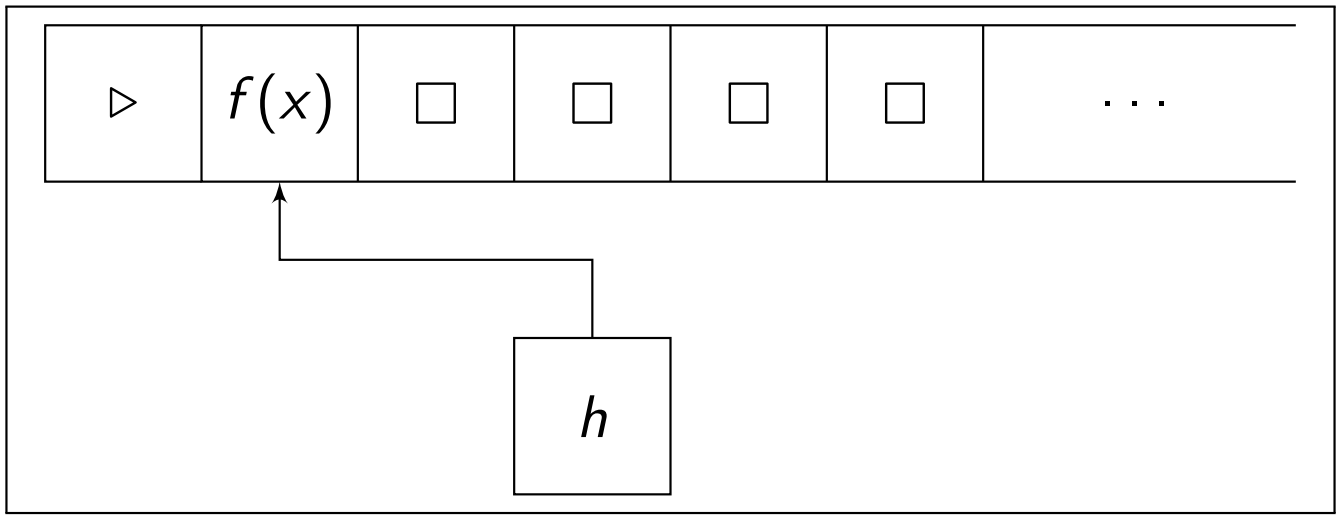
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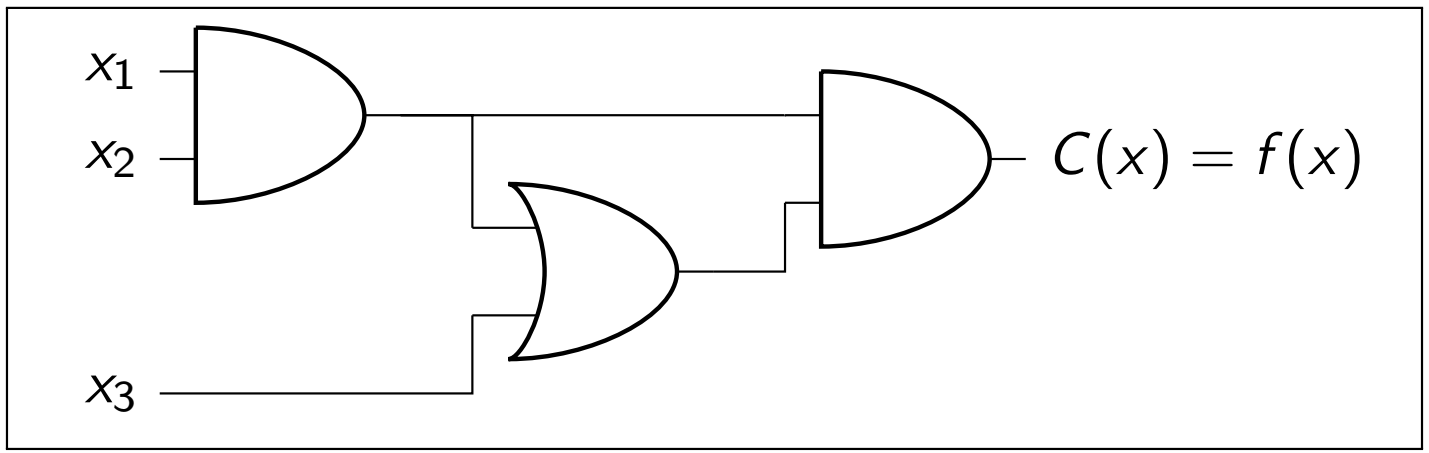

x_1
x_2
x_3
$C(x) = f(x)$

Definition

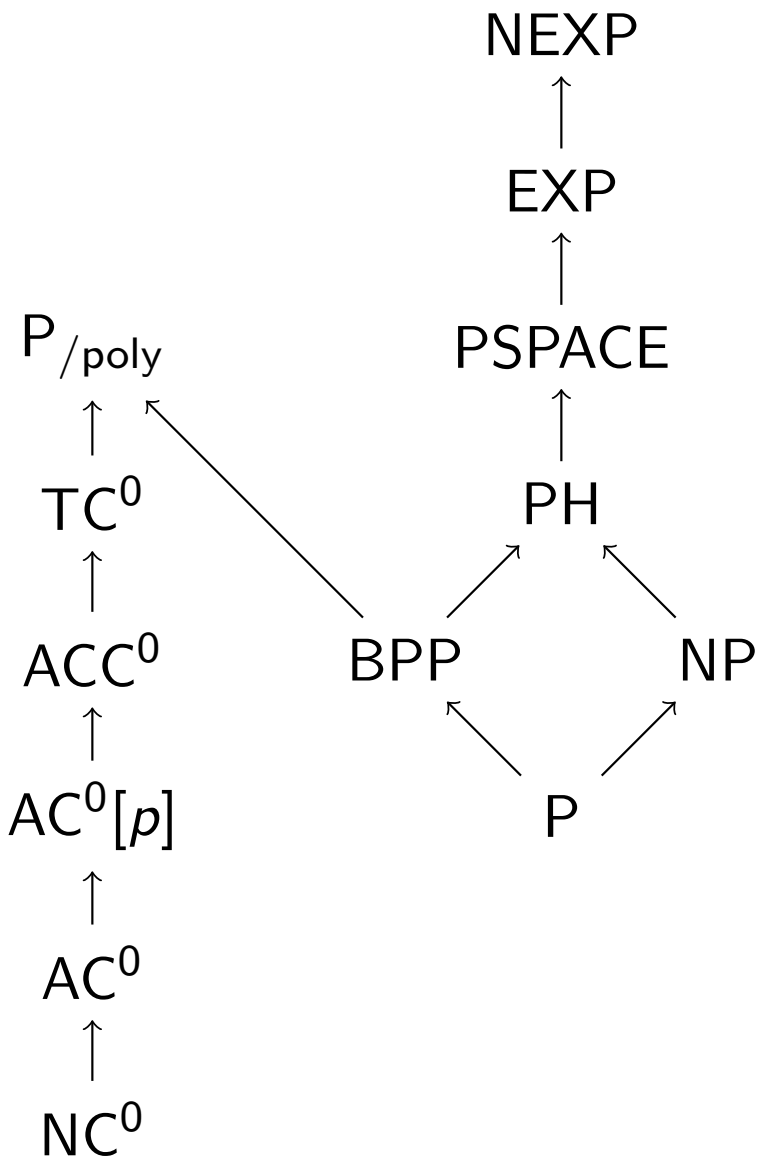
$\mathsf{P} = \bigcup_{k \in \mathbb{N}} \mathsf{TIME}(n^k)$.

Definition

$\mathsf{P}_{/\mathsf{poly}} = \bigcup_{k \in \mathbb{N}} \mathsf{SIZE}(n^k)$.

Lemma

$\mathsf{P} \subsetneq \mathsf{P}_{/\mathsf{poly}}$.


NEXP
EXP
PSPACE
PH
NP
P
BPP
$P_{/poly}$
TC^0
ACC^0
$AC^0[p]$
AC^0
NC^0

Circuit Lower Bounds

Question (OPEN)

$\mathsf{NP} \overset{?}{\subseteq} \mathsf{P}$.

Question (OPEN)

$\mathsf{NP} \overset{?}{\subseteq} \mathsf{P}_{/\mathsf{poly}}$.

Question

$\mathsf{NP} \stackrel{?}{\subseteq} \mathsf{NC}^0$.

Lemma

$\mathsf{NP} \not\subseteq \mathsf{NC}^0$.

Question

$\mathsf{NP} \stackrel{?}{\subseteq} \mathsf{AC}^0$.

Theorem ([FSS81, Ajt83, Yao85, Hås86])

$\oplus \notin \mathsf{AC}^0$.

Corollary

$\mathsf{NP} \not\subseteq \mathsf{AC}^0$.

Question

$\mathsf{NP} \stackrel{?}{\subseteq} \mathsf{AC}^0[p]$.

Theorem ([Raz87, Smo87])

$\mathtt{MOD}_q \notin \mathsf{AC}^0[p]$.

Corollary

$\mathsf{NP} \not\subseteq \mathsf{AC}^0[p]$.

Question (OPEN)

$\mathsf{NP} \stackrel{?}{\subseteq} \mathsf{ACC}^0$.

Theorem ([MW18])

$\mathsf{NQuasiP} \not\subseteq \mathsf{ACC}^0$.

Circuit Analysis Notions

Definition (Satisfiability)

Input : A Boolean circuit $C : \{0,1\}^n \to \{0,1\}$ from a circuit class $\mathfrak{C}$.

Question : Is there any $x \in \{0,1\}^n$ such that $C(x) = 1$?

Definition (Minimum Circuit Size Problem [KC00])

Input : The truth table $\mathtt{tt}(f) \in \{0,1\}^{2^n}$ of a Boolean function $f : \{0,1\}^n \to \{0,1\}$ and an integer $1 \leq s \leq 2^n$.

Question : Is there any Boolean circuit $C : \{0,1\}^n \to \{0,1\}$ of size $|C| \leq s$ that computes f?

Definition (Learning [Val84, KV94])

Let $0 < \varepsilon, \delta < 1$.

Input : An oracle for some Boolean function $f : \{0,1\}^n \to \{0,1\}$ from a circuit class $\mathfrak{C}$.

Output : A circuit $h : \{0,1\}^n \to \{0,1\}$, such that

$$\Pr_{x \sim \{0,1\}^n}[h(x) \neq f(x)] \leq \varepsilon,$$

with probability at least $1 - \delta$.

Definition (Compression [CKK+14])

Input : The truth table $\mathtt{tt}(f) \in \{0,1\}^{2^n}$ of some Boolean function $f : \{0,1\}^n \to \{0,1\}$ from a circuit class $\mathfrak{C}$.

Output : A circuit C of size $o(2^n/n)$ that computes f.

Definition (Natural Properties [RR94])

Let

- $\mathcal{F}_n$ denote the set of all Boolean functions on n variables,
- D be some complexity class, and
- $s : \mathbb{N} \to \mathbb{N}$ is a size function.

A D-natural property useful against $\mathfrak{C}[s]$ is an algorithm $A : \{0,1\}^{2^n} \to \{0,1\}$ such that

1. A is computable in D,
2. if $f \in \mathfrak{C}[s]$, then $A(\mathtt{tt}(f)) = 0$, and
3. $\Pr_{g \sim \mathcal{F}_n}[A(\mathtt{tt}(g)) = 1] \geq \frac{1}{2}$.

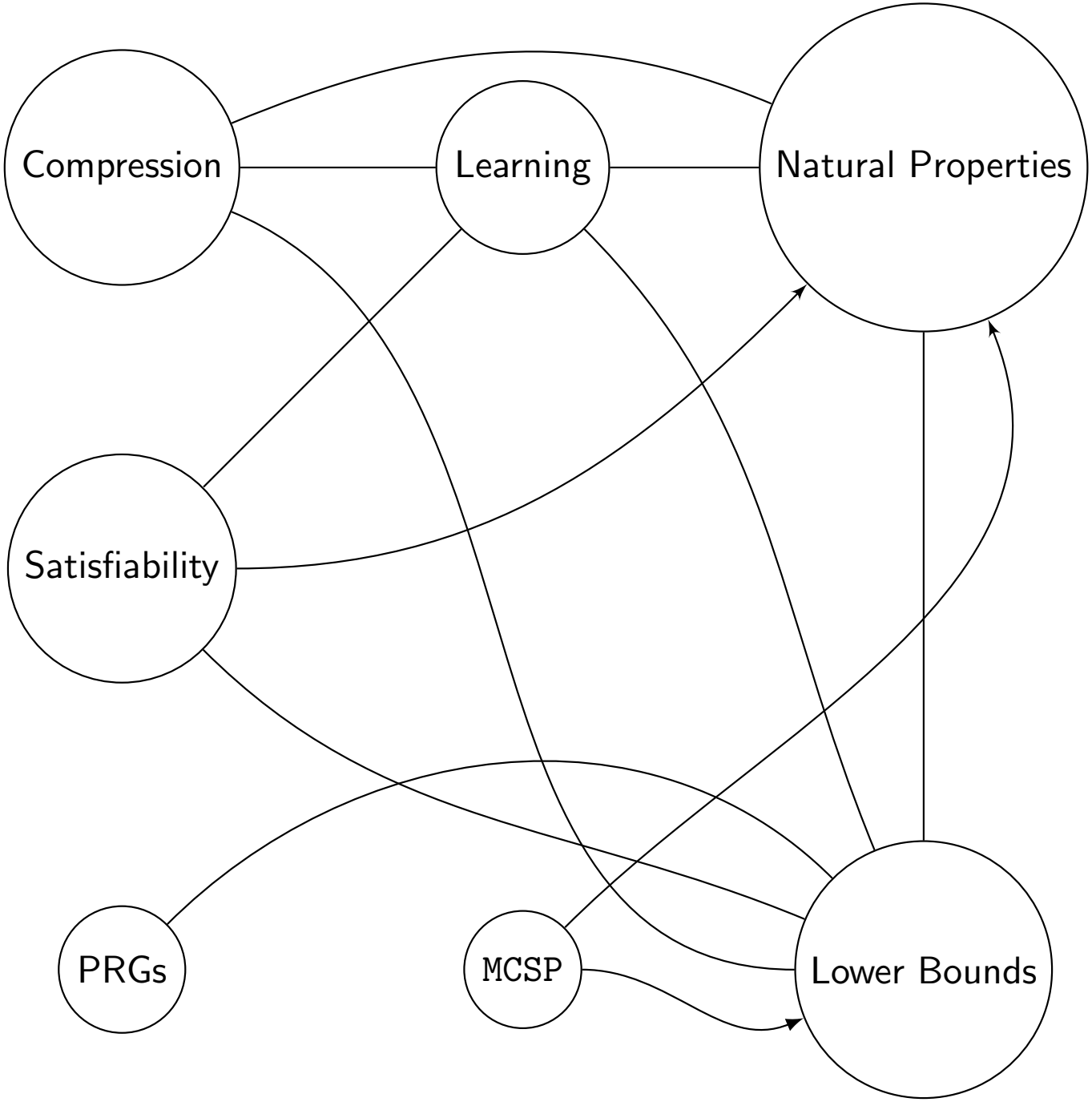
Definition (PRGs)

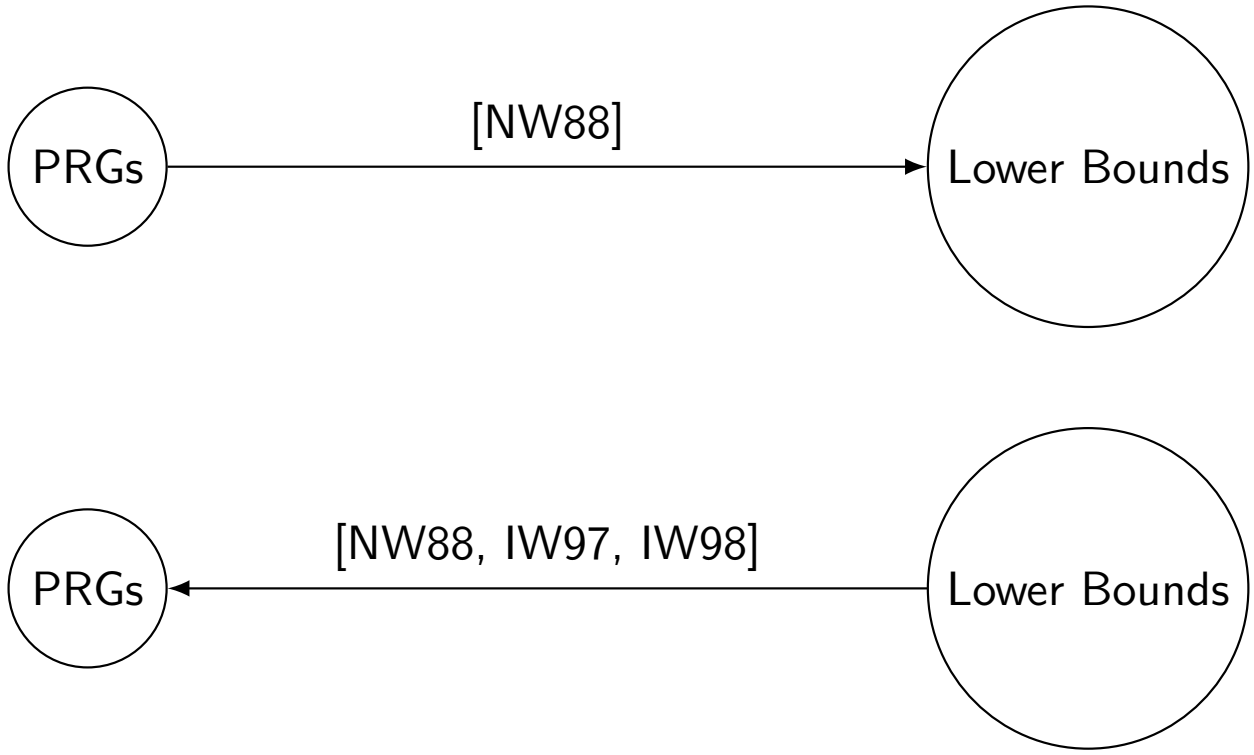
Let $\mathcal{F}$ be a class of functions on n variables and $\varepsilon > 0$. We say that $G : \{0,1\}^s \to \{0,1\}^n$ is a PRG that ε-fools $\mathcal{F}$ if

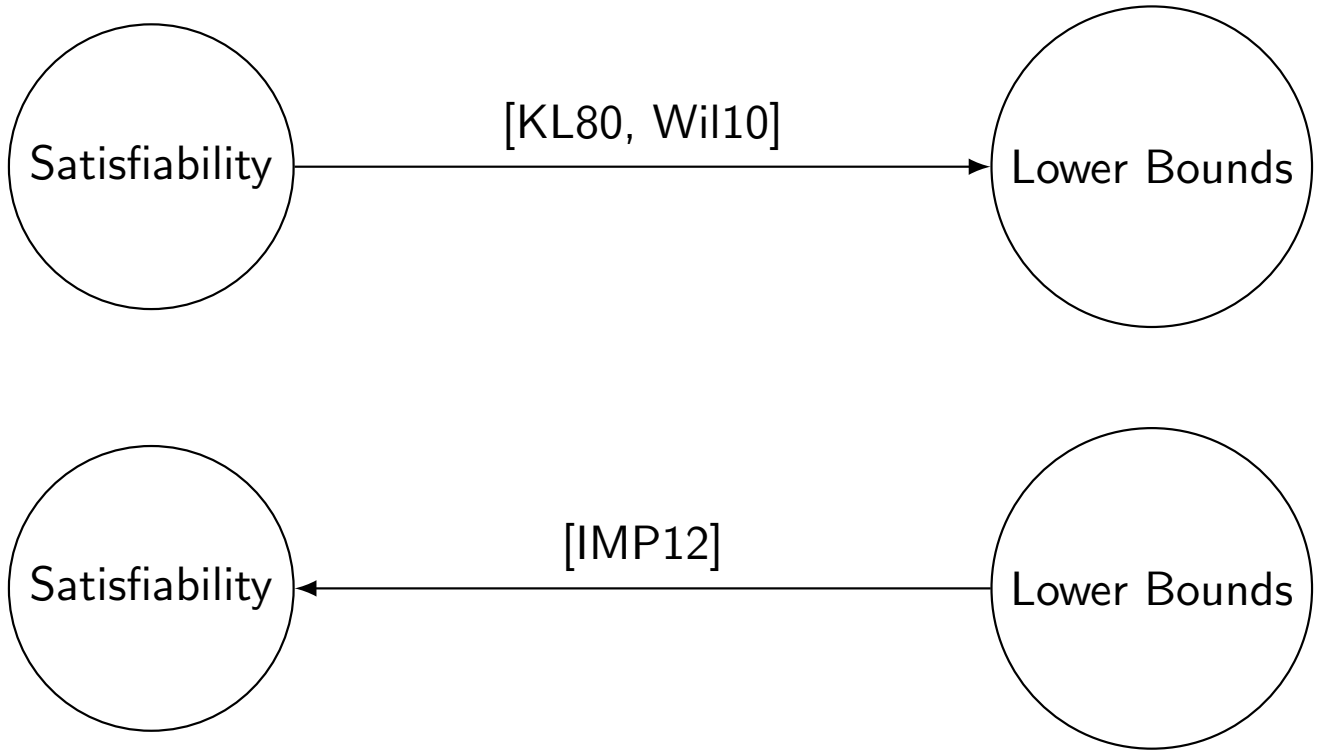
$$\left| \Pr_{y \sim \{0,1\}^s}[f(G(y)) = 1] - \Pr_{x \sim \{0,1\}^n}[f(x) = 1] \right| \leq \varepsilon,$$

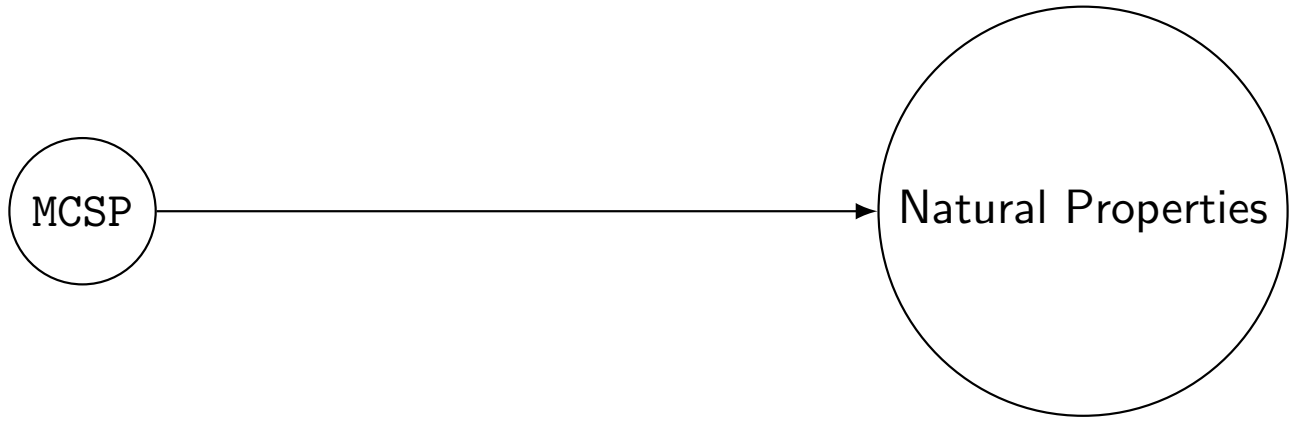
for all $f \in \mathcal{F}$.

Known Connections

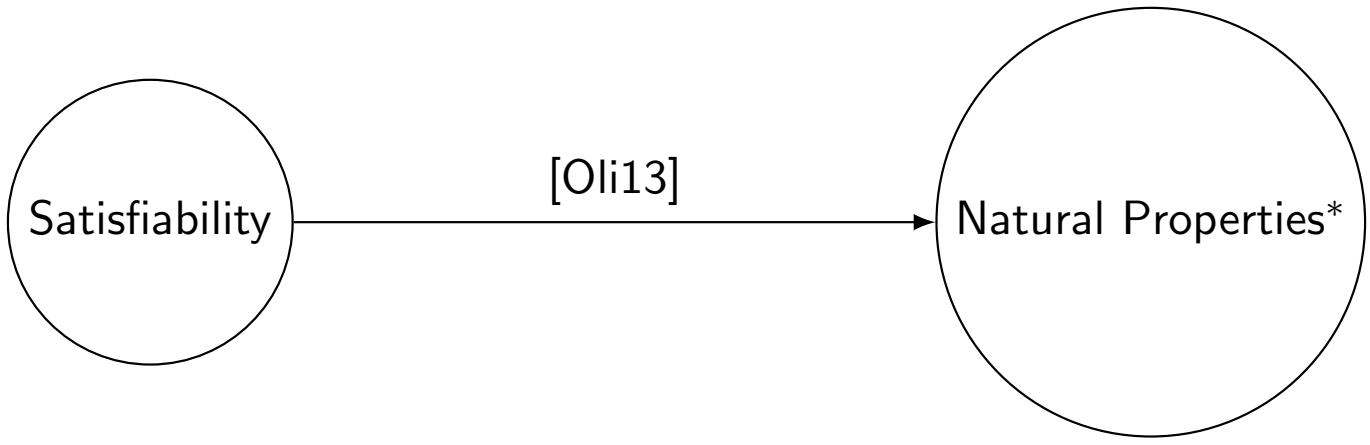

Compression
Learning
Natural Properties
Satisfiability
PRGs
MCSP
Lower Bounds

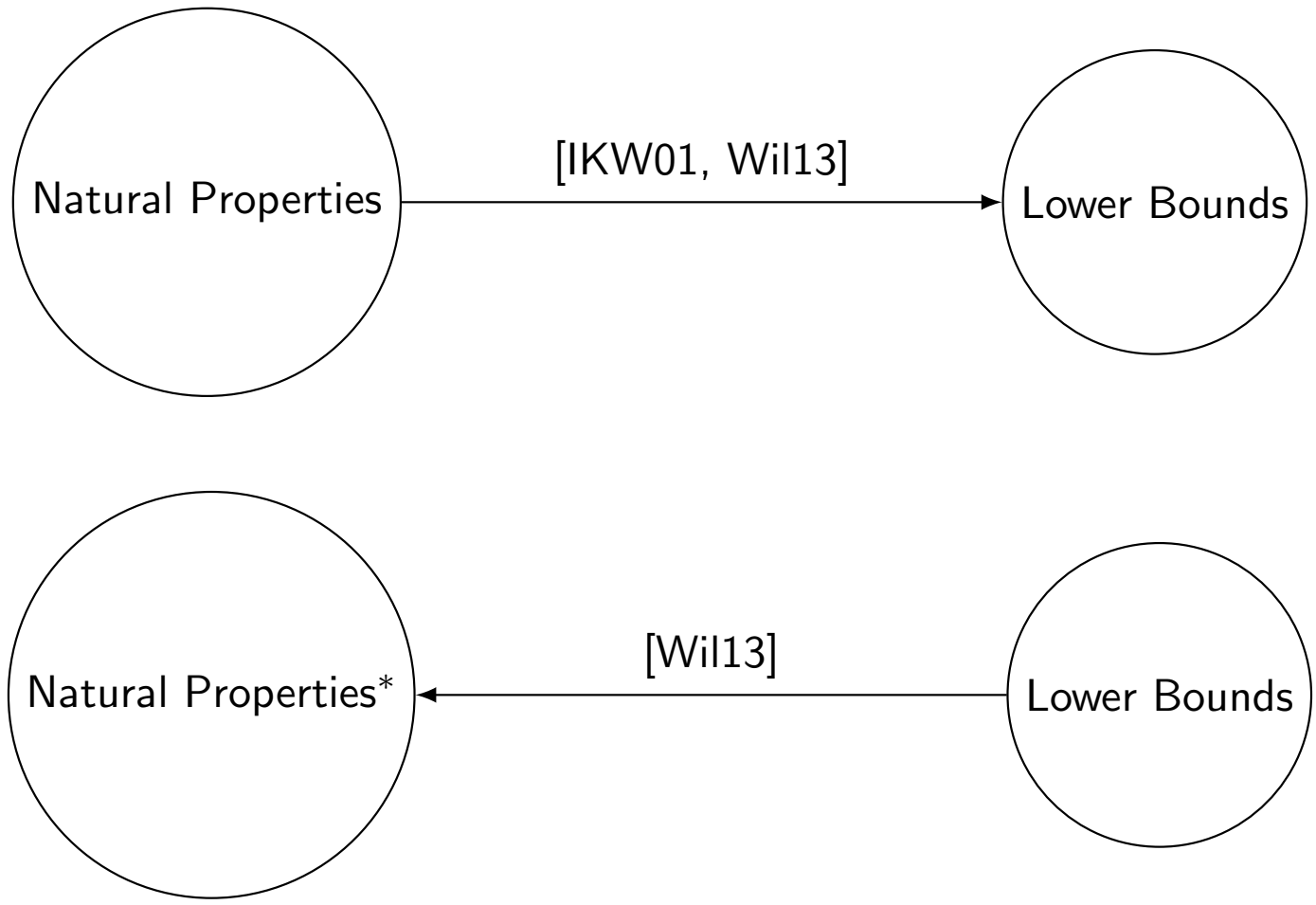

PRGs
[NW88]
Lower Bounds
PRGs
[NW88, IW97, IW98]
Lower Bounds

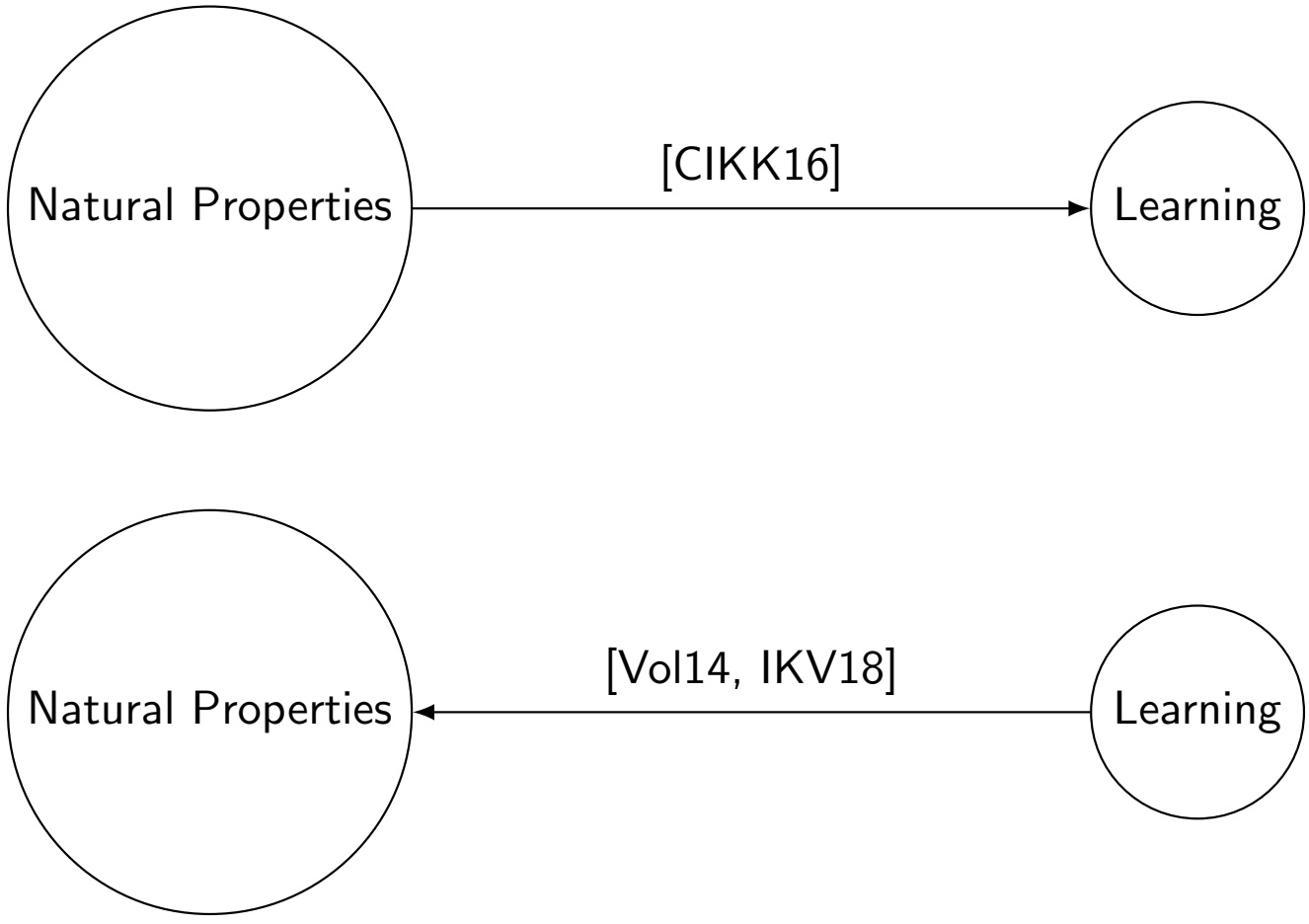

Satisfiability
[KL80, Wil10]
Lower Bounds
Satisfiability
[IMP12]
Lower Bounds

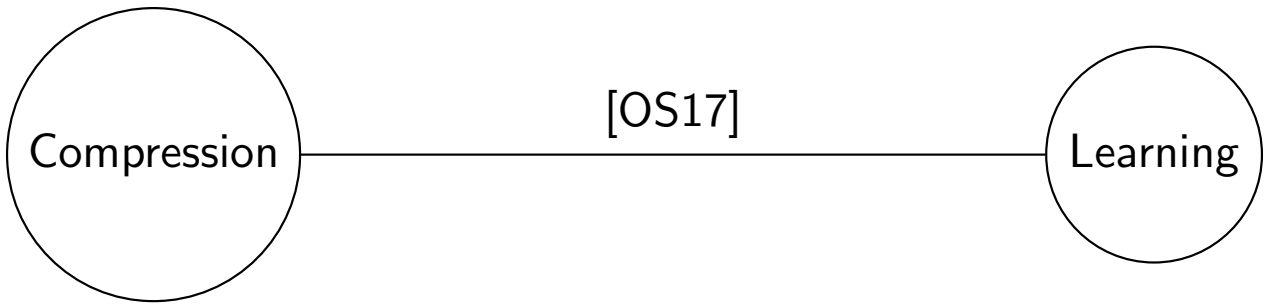

MCSP
Natural Properties

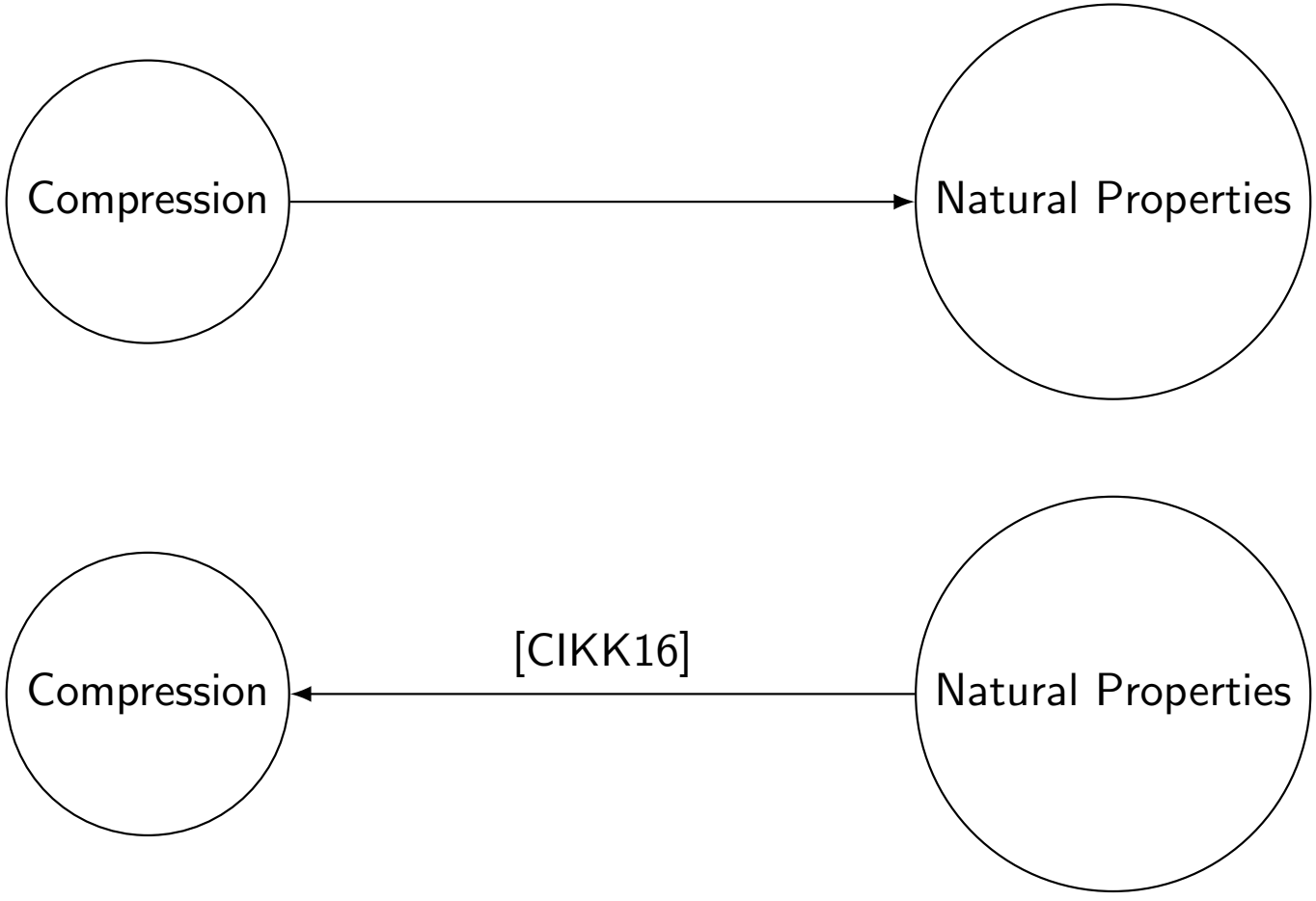

Natural Properties
Natural Properties*

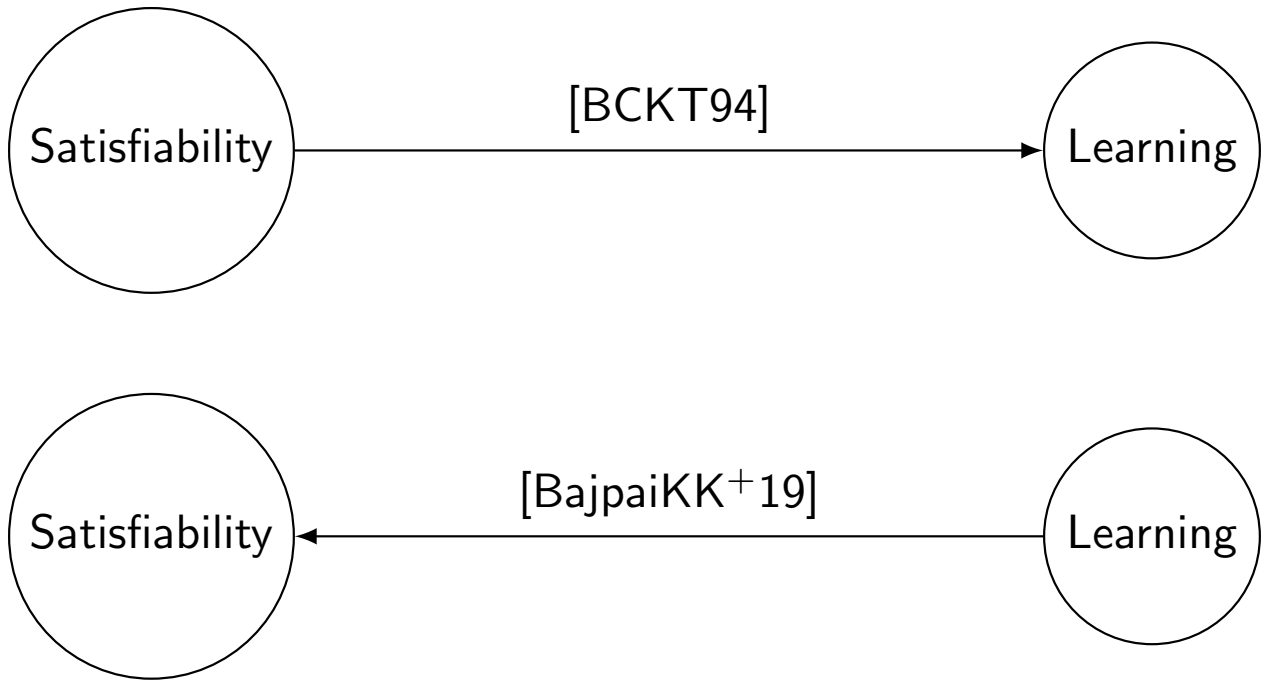

Satisfiability
[Oli13]
Natural Properties*

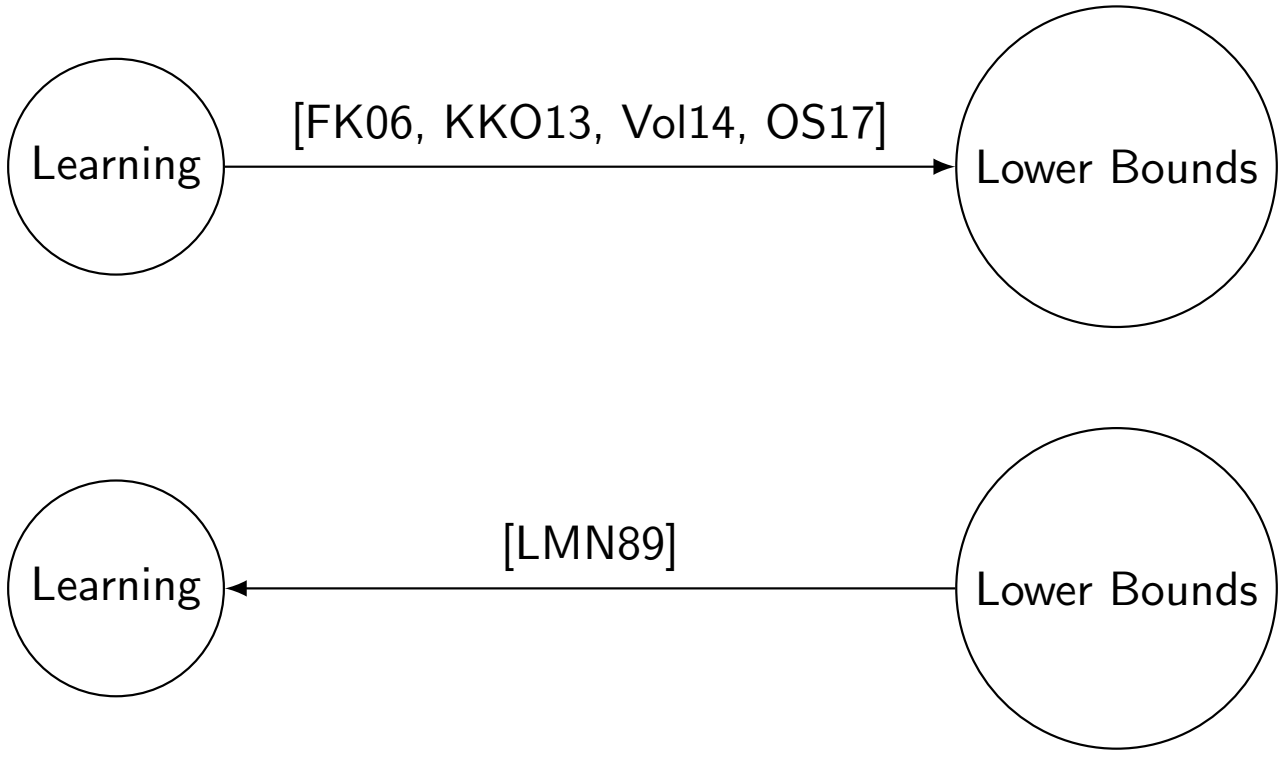

Natural Properties
[IKW01, Wil13]
Lower Bounds
Natural Properties*
[Wil13]
Lower Bounds

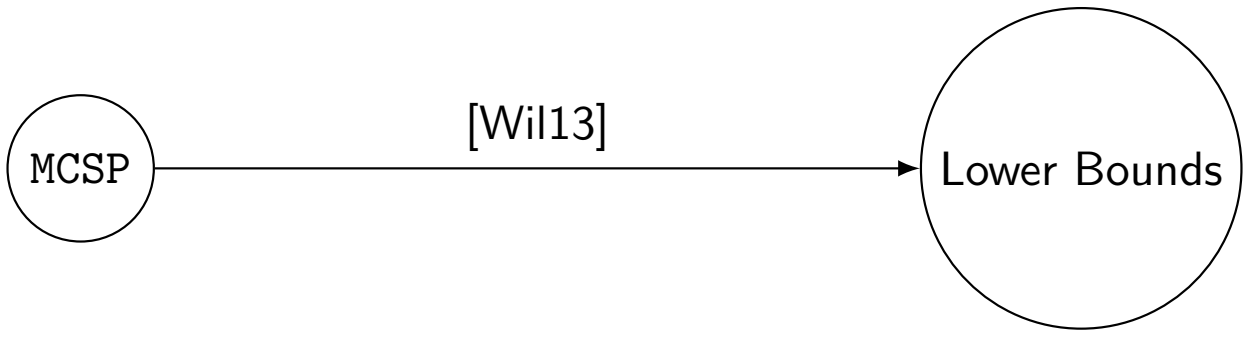

Natural Properties
[CIKK16]
Learning
Natural Properties
[Vol14, IKV18]
Learning

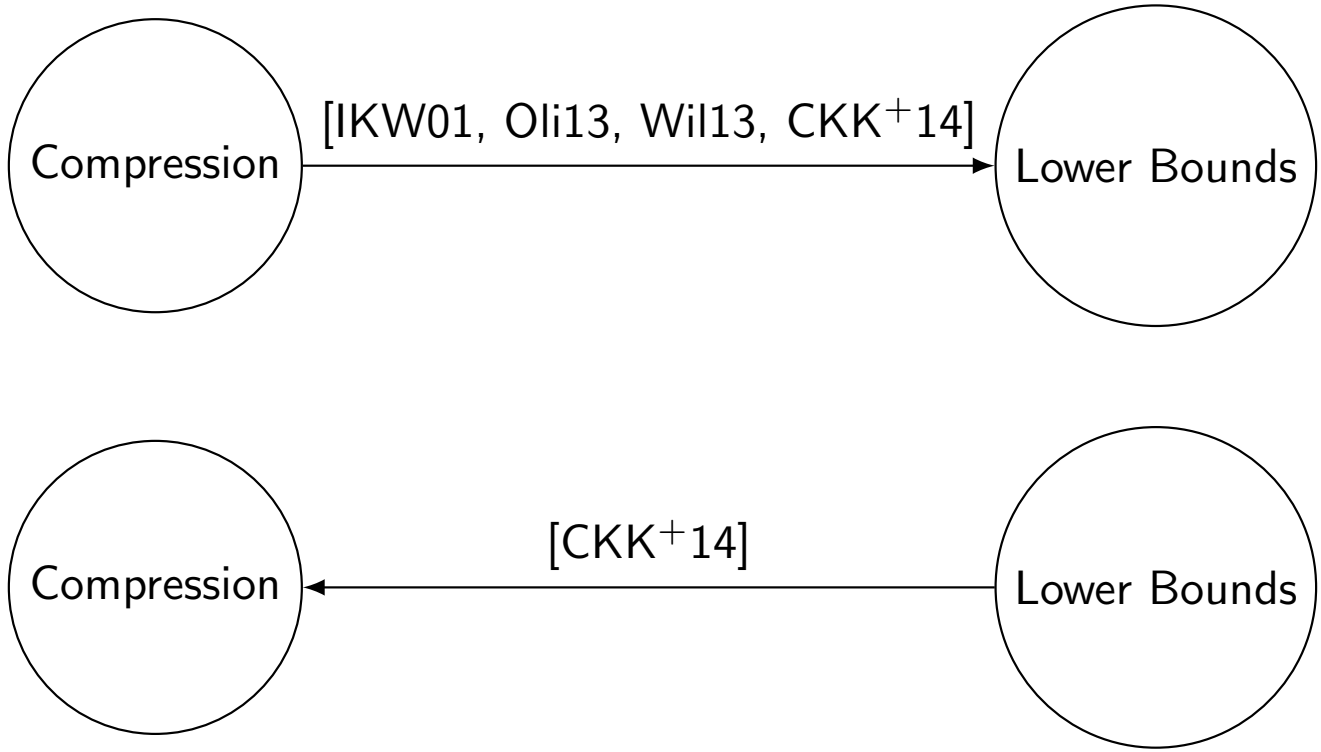

Compression
[OS17]
Learning


Compression
Natural Properties
Compression
[CIKK16]
Natural Properties

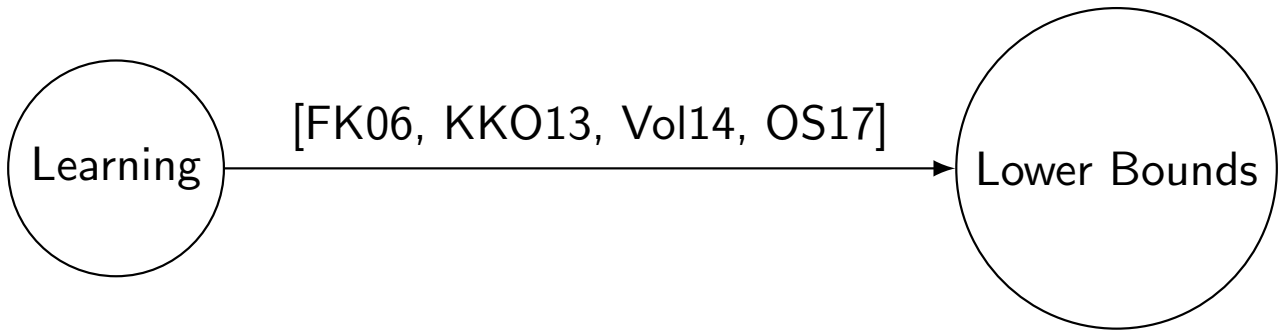

Satisfiability
[BCKT94]
Learning
Satisfiability
[BajpaiKK+19]
Learning

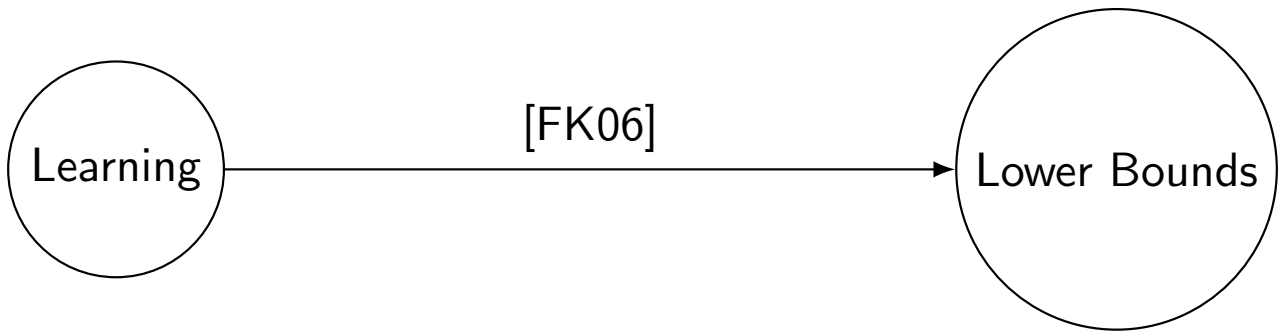

Learning
[FK06, KKO13, Vol14, OS17]
Lower Bounds
Learning
[LMN89]
Lower Bounds


MCSP
[Wil13]
Lower Bounds


Compression
[IKW01, Oli13, Wil13, CKK+14]
Lower Bounds
Compression
[CKK+14]
Lower Bounds

Example: Learning to Lower Bounds


Learning
[FK06, KKO13, Vol14, OS17]
Lower Bounds


Learning
[FK06]
Lower Bounds

Theorem ([FK06])

Let $s : \mathbb{N} \to \mathbb{N}$ *be a (polynomially-bounded) size function.*

Let $\mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$ *be a circuit class exactly learnable in time* $t := 2^{s^{o(1)}}$ *from membership and equivalence queries.*

Then, $\mathsf{EXP}^{\mathsf{NP}} \not\subseteq \mathfrak{C}[s]$.

Note: The learning algorithm runs in subexponential time.

Definition

Let $f \in \mathfrak{C}[s]$ with $f : \{0,1\}^n \to \{0,1\}$.

$$\forall x \in \{0,1\}^n : \mathtt{MQ}(x) = f(x).$$

Definition

Let $f \in \mathfrak{C}[s]$ with $f : \{0,1\}^n \to \{0,1\}$.

For all hypotheses (circuits) $h : \{0,1\}^n \to \{0,1\}$ we have that

$$\mathtt{EQ}(h) = \begin{cases} 1, & \text{if } h(x) = f(x) \text{ for all } x \in \{0,1\}^n, \\ y : h(y) \neq f(y), & \text{otherwise.} \end{cases}$$

Theorem ([FK06])

Let $s : \mathbb{N} \to \mathbb{N}$ *be a (polynomially-bounded) size function.*

Let $\mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$ *be a circuit class exactly learnable in time* $t := 2^{s^{o(1)}}$ *from membership and equivalence queries.*

Then, $\mathsf{EXP}^{\mathsf{NP}} \not\subseteq \mathfrak{C}[s]$.

Note: The learning algorithm runs in subexponential time.

Proof.

1. Assume that $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$. $(*)$
2. This implies $\mathsf{EXP}^{\mathsf{NP}} = \mathsf{P}^{\#\mathsf{P}}$.
3. Thus $\mathtt{PERM}$ is complete for $\mathsf{EXP}^{\mathsf{NP}}$.
4. Now use the (time-t) learning algorithm to show

$$\mathtt{PERM} \in \mathsf{SUBEXP}^{\mathsf{NP}} := \mathsf{TIME}\left(2^{n^{o(1)}}\right)^{\mathsf{NP}}.$$

5. This yields $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathsf{SUBEXP}^{\mathsf{NP}}$. A contradiction!

Proof.

1. Assume that $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$. $(*)$
2. **This implies** $\mathsf{EXP}^{\mathsf{NP}} = \mathsf{P}^{\#\mathsf{P}}$**.**
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Proof (continued).

Assume that $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$.

This implies $\mathsf{EXP}^{\mathsf{NP}} = \mathsf{P}^{\#\mathsf{P}}$, as

$$\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathsf{P}_{/\mathsf{poly}} \overset{[\mathsf{BH92}]}{\Longrightarrow} \mathsf{EXP}^{\mathsf{NP}} = \mathsf{EXP},$$

$$\mathsf{EXP} \subseteq \mathsf{P}_{/\mathsf{poly}} \overset{[\mathsf{BN91}]}{\Longrightarrow} \mathsf{EXP} = \mathsf{MA},$$

and

$$\mathsf{MA} \subseteq \mathsf{PH} \subseteq \mathsf{P}^{\#\mathsf{P}}.$$

Proof (continued).

1. Assume that $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$. $(*)$
2. This implies $\mathsf{EXP}^{\mathsf{NP}} = \mathsf{P}^{\#\mathsf{P}}$.
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Proof (continued).

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2. This implies $\mathsf{EXP}^{\mathsf{NP}} = \mathsf{P}^{\#\mathsf{P}}$.
3. Thus $\mathtt{PERM}$ is complete for $\mathsf{EXP}^{\mathsf{NP}}$.
4. **Now use the (time-t) learning algorithm to show**

$$\mathtt{PERM} \in \mathsf{SUBEXP}^{\mathsf{NP}} := \mathsf{TIME}\left(2^{n^{o(1)}}\right)^{\mathsf{NP}}.$$

5. This yields $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathsf{SUBEXP}^{\mathsf{NP}}$. A contradiction!

Proof (continued).

We will use the (time-t) learning algorithm to show

$$\texttt{PERM} \in \mathsf{SUBEXP}^{\mathsf{NP}},$$

by induction on i.

Here, i refers to the input matrices size, namely $i \times i$.

Proof (continued).

Input: x; a $n \times n$ Boolean matrix.

For $i \leftarrow 1$ to n:

a. If $i = 1$, then output the trivial circuit for $\texttt{PERM}$ on 1×1 matrices; else

b. Run the (time-t) exact learning algorithm to find C_i, the circuit that computes $\texttt{PERM}$ on $i \times i$ matrices.

c. Simulate $\texttt{MQ}$ and $\texttt{EQ}$ using C_{i-1} and an NP oracle.

Output: $C_n(x) = \texttt{PERM}(x)$.

Proof (continued).

Input: x; a $n \times n$ Boolean matrix.

For $i \leftarrow 1$ to n:

a. If $i = 1$, then output the trivial circuit for $\texttt{PERM}$ on 1×1 matrices; else

b. Run the (time-t) exact learning algorithm to find C_i, the circuit that computes $\texttt{PERM}$ on $i \times i$ matrices.

c. **Simulate $\texttt{MQ}$ and $\texttt{EQ}$ using C_{i-1} and an NP oracle.**

Output: $C_n(x) = \texttt{PERM}(x)$.

Proof (continued).

Simulate $\texttt{MQ}(y) = \texttt{PERM}(y)$, with $|y| = i$, using C_{i-1}.

Self-reduction: $\texttt{PERM}(y)$ can be computed by i calls to C_{i-1}.

This runs in polynomial (in n) time as $i \leq n$ and C_{i-1} is of polynomial (in n) size.

Proof (continued).

Simulate $\texttt{EQ}(h)$, with $h : \{0,1\}^i \to \{0,1\}$, using C_{i-1} and an NP oracle:

i. Ask the NP oracle:

> *"Does there exist a* $z \in \{0,1\}^i$ *such that* $h(z) \neq \texttt{PERM}(z)$*?"*

ii. If NO, then $h(x) = \texttt{PERM}(x)$ for all $x \in \{0,1\}^i$.

iii. If YES, then find z. Ask the NP oracle:

> *"Does there exist a* $z \in 0\{0,1\}^{i-1}$ *such that* $h(z) \neq \texttt{PERM}(z)$*?"*

Repeat until all i bits of z are computed.

This runs in polynomial (in n) time as $i \leq n$ and h and C_{i-1} are of polynomial (in n) size.

Proof (continued).

Input: x; a $n \times n$ Boolean matrix.

For $i \leftarrow 1$ to n:

a. If $i = 1$, then output the trivial circuit for $\texttt{PERM}$ on 1×1 matrices; else

b. Run the (time-t) exact learning algorithm to find C_i, the circuit that computes $\texttt{PERM}$ on $i \times i$ matrices.

c. Simulate $\texttt{MQ}$ and $\texttt{EQ}$ using C_{i-1} and an NP oracle.

Output: $C_n(x) = \texttt{PERM}(x)$.

Proof (continued).

What is the running time?

The running time of the previous procedure is

$$\mathrm{poly}(n, t) = \mathrm{poly}\left(n, 2^{s^{o(1)}}\right).$$

Note that $s = \mathrm{poly}(n)$; this yields $\texttt{PERM} \in \mathrm{SUBEXP}^{\mathrm{NP}}$.

Proof.

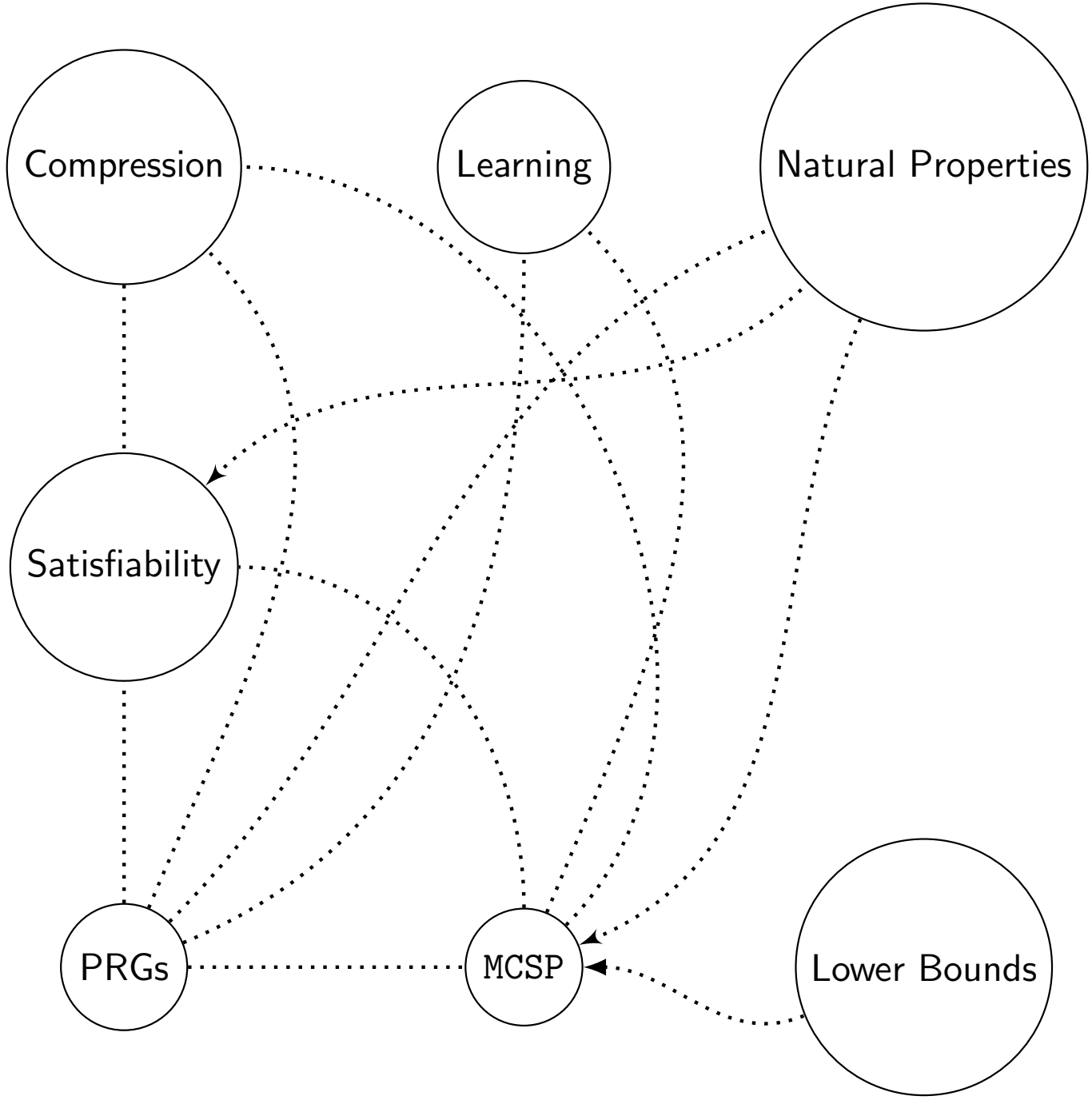
1. Assume that $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathfrak{C}[s] \subseteq \mathsf{P}_{/\mathsf{poly}}$. $(*)$
2. This implies $\mathsf{EXP}^{\mathsf{NP}} = \mathsf{P}^{\#\mathsf{P}}$.
3. Thus $\mathtt{PERM}$ is complete for $\mathsf{EXP}^{\mathsf{NP}}$.
4. Now use the (time-t) learning algorithm to show

$$\mathtt{PERM} \in \mathsf{SUBEXP}^{\mathsf{NP}} := \mathsf{TIME}\left(2^{n^{o(1)}}\right)^{\mathsf{NP}}.$$

5. This yields $\mathsf{EXP}^{\mathsf{NP}} \subseteq \mathsf{SUBEXP}^{\mathsf{NP}}$. A contradiction!

The proof is complete. □

Discussion: OPEN Problems


Compression
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Acknowledgements I

Many thanks to Igor Oliveira (Oxford) and Ryan Williams (MIT).

Acknowledgements II

Many thanks to Valentine Kabanets (SFU), Rahul Santhanam (Oxford), and Igor Oliveira (Oxford).

Thank you!

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